The Use of Computational Tools within Mathematical Work Practices

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ABSTRACT

There has been little investigation of how professional mathematicians use computational tools for mathematics within their work practices. We overview a pair of qualitative studies examining the role of computational tools within the workflow of twenty professional mathematicians in a research setting. While these studies revealed a number of ways in which the interaction design of such tools is failing to integrate with existing work practices, moving to the designs of improved versions presents a number of challenges. These challenges pertain to interfacing with those who possess the mathematical expertise necessary to build functional tools and whether or not improved computational tools should be pursued as replacements to physical media. We discuss these challenges and present some preliminary ideas for solutions.

Author Keywords

mathematical problem solving, Computer Algebra Systems, qualitative studies, contextual analysis

ACM Classification Keywords

H.5.2 Information Interfaces and Presentation: Miscellaneous

INTRODUCTION

Over the past 30 years, significant effort has been devoted to designing computational tools to support mathematical problem solving. Results of this work have produced a number of tools (e.g., Maple, Mathematica and Matlab) which are used by hundreds of thousands of people every year [1]. Despite the widespread adoption of these tools, there has been little contextual-analysis style research to understand how they are used in practice. Consequently, the community is lacking in-depth descriptions of how mathematicians work, how existing computational tools fit into mathematicians' workflow, and the strengths and limitations of computational tools as compared to more traditional physical media such as paper and whiteboards. Such information is crucial to the design of future versions.

Research that has examined mathematical work practices has tended to do so in educational settings (e.g., [7], [9], [10]). In these studies, students are using computational tools to solve well-defined, instructor-supplied problems, with the goal of learning rather fundamental mathematical concepts. Other work has focused on examining broadly applicable usability issues within laboratory settings, such as the ease with which users can enter expressions using different input devices or modalities (e.g., [2], [8]).

In our work, we sought to understand the extent to which existing computational tools for mathematics support *professional* work practices. To this end, we conducted two qualitative studies with expert mathematicians in a research setting. The first was with a group of nine theoretical mathematics [3]. The second was with 11 experts in more applied fields with a strong mathematical component (e.g., engineering and physics). Both groups are solving problems that are largely ill-defined. In the case of the theoretical mathematicians, it is with the goal of gaining new mathematical knowledge, whereas the more applied researchers use mathematics to gain new insight into other scientific phenomena.

Our study revealed a number of ways in which the interaction design of existing computational tools is failing to support professional mathematical work practices (described briefly in upcoming sections). Therefore, a next logical step in the research process would be to design an improved computational tool for mathematical problem solving. Proceeding to a newly designed tool, however, presents a number of challenges. The first set of challenges involves pragmatic issues. Any re-design requires both communication with and buy-in from those capable of building the mathematical problem-solving engines, i.e., people with a highly sophisticated level of mathematics expertise. Challenges that accompany this type of collaboration include how to communicate our findings in a sufficiently compelling manner for this audience and how to reconcile diverging research or commercial agendas. We are also faced with a challenge that is more philosophical in nature. While our study uncovered limitations of existing computational tools, the extent to which current work practices, which rely heavily on physical media, require "improving" is debatable. Thus, the question

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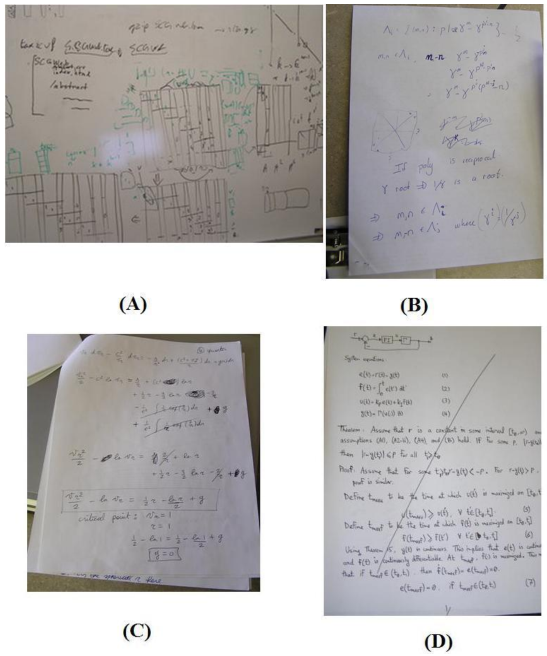

Figure 1. A subset of the timeline showing the work progressing from a rough brainstorming phase (A) to a formal narrative (D).

Figure 2. A subset of the affinity diagrams

is what should happen within the HCI community when the results of contextual analysis do not lend themselves to improved design?

The remainder of this paper is structured as follows. To set the context, we begin by describing our study methodology and data analysis techniques. We next provide a brief overview of our findings. We then describe the challenges we face in moving to design and discuss our preliminary ideas for addressing some of these challenges.

STUDY METHOD

In both studies, our participants consisted of professional mathematicians doing research within a university setting. In the first study, all nine participants do theoretical work in the Applied Math, Pure Math and Computer Science departments. In the second study, we widened our sample to include 11 participants who work on more applied mathematical problems in disciplines such as Physics, Mechanical Engineering and Electrical and Computer Engineering. All participants had advanced mathematical knowledge, with 17 of the 20 participants at the Ph.D. level or higher.

To understand our participants' work practices, we conducted semi-structured interviews, which sought to answer the following questions:

- What is the goal of the mathematicians? What are they seeking to accomplish or "produce"?
- What characterizes the mathematicians' workflow?
- Which tools are used in mathematical problem solving (e.g., paper, whiteboard, Computer Algebra Systems (CAS)), at which points in the work process, and for what reasons?
- What types of tasks are best supported by the different tools and why?

Interviews took place at locations identified by participants as their primary workspace (either offices or labs), enabling us to view and document the work environments and artifacts. During the interviews, which lasted approximately 30-45 minutes each, we asked participants to describe their research practices. To ground the interviews and assist with recall, we asked participants to walk us through specific instances of recent research work. We also asked participants to show us samples of their work. These samples both provided invaluable insights into the nature of their work and prompted a great deal of detailed discussion on the manner in which they solve mathematical problems.

Data collection included audio tapes of the interview sessions and digital photographs of the work artifacts and environments.

ANALYSIS TECHNIQUES

Our data analysis techniques and the output of these techniques were as follows. First, we transcribed the audio tapes of the interview sessions. Next, we created two types of affinity diagrams: one for the interview statements and another for components of digital photographs. Figure 2 shows a small subset of the affinity diagrams for interview statements from the first study alone. We also created sequence models, which enabled us to abstract and label key phases of our participants' workflow. Finally, when creating the affinity diagrams for the digital photos, we began to see a temporal progression in the work starting from a rough initial state to a formal narrative. Thus, we pooled all participants' photographs and into one "timeline" of work. Figure 1 shows a subset of this timeline.

OVERVIEW OF FINDINGS

The above analysis techniques resulted in a number of findings, which we summarize briefly here and refer the reader to [3] for more detail.

Goal/Product of the Work

For the more theoretical mathematicians, the goal is to create a formal mathematical narrative. We also found that a central part of the mathematicians' workflow is to *evolve* this narrative from a rough initial state to its final form, and that this evolution often takes place almost entirely within physical media. For more applied researchers, less emphasis is placed on creating a narrative and more on using the mathematical results to describe or demonstrate other scientific phenomena.

Key Roles of Computational Tools

Computational tools are used by the professional mathematicians, but in a more limited manner than we had anticipated. Instead of using the power of tools such as Maple to solve complex problems efficiently, the work artifacts and interview statements revealed that participants rely heavily on physical media for their work. When they do use computational tools, it is mainly to solve long, tedious expressions that cannot be computed by hand or to verify hand-derived work. While still relying heavily on physical media, applied mathematicians do seem to make greater use of computational tools than the theoretical researchers, with an additional popular use case being for simulation.

Open Issues for the Design of Computational Tools

We identified three key themes affecting the utility of computational tools for advanced mathematical problem solving, each of which has corresponding implications for design:

1. *The need for transparency in the tools' reasoning:* Most participants feel they gain more insight when doing the problems by hand, suggesting a lack of transparency within existing computational tools. In addition to limiting insight, this lack of transparency causes many participants to have difficulty trusting the computational tools.

2. *The need for free-form 2D representations:* Mathematical problem-solving and, in particular, narrative creation relies heavily on 2D representations, such as diagrams, symbols and prose. Physical media provide the ability to manipulate these representations *in-place* as the work evolves. Current computational tools, on the other hand, tend to support only a rigid, formal, input-output style dialog.

3. *The need for collaboration support:* Mathematics research, particularly in early stages of problem solving, is a highly collaborative activity. Current computational tools do little to facilitate close collaboration.

MOVING TO DESIGN: CHALLENGES

As described above, our study revealed a number of ways in which computational tools are limited, in turn suggesting a number of potential implications for design. Moving forward with this re-design, however, presents three main challenges. The first two relate to our need to collaborate with those with the mathematical expertise necessary to build or integrate the mathematical back-ends. The third concerns the extent to which current work practices, which rely heavily on physical media, actually need improving.

Communication

A first challenge concerns communication. Creating new Computer Algebra System (CAS) prototypes, given the extent and sophistication of the math reasoning, is not something that we as HCI researchers are able to undertake without tight collaboration with those with a strong mathematics background. A primary challenge with this collaboration is presenting evidence of work practices in a sufficiently compelling manner. Qualitative research is not something that many mathematicians are familiar with. Our experience is that the output of contextual analysis (e.g., themes with supporting quotes, the affinity diagrams, work models and timelines) does not necessarily serve as sufficient evidence for this group of individuals, who are accustomed to more concrete, black-and-white, and quantitative research output.

Diverging Agendas

In addition to communication, a related issue when collaborating with mathematicians or developers capable of creating new prototypes pertains to diverging agendas. Research problems that a symbolic computation researcher is interested in tackling, for example, do not necessarily match the implications for design that arose from our work. For collaboration with industrial patterns, there is the issue of competitive advantage. One of the key usability issues identified from our study is a lack of transparency. Companies, however, might be reluctant to release all of key details of their underlying algorithms as these algorithms are part of what sells their product.

A promising avenue for collaboration is with researchers in the area of pen-math, who are exploring the use of pen-based input to computational mathematical engines (e.g., [5], [6],). However, while pen-based interaction has great potential to allow for more free-form narrative construction, the focus of this research is mainly on recognition accuracy, dialogs to repair mis-recognized expressions, and ways to present manipulation possibilities once the expression has been recognized. Given the complexity of these basic issues, they are largely being explored within the context of fairly strict input-output style dialogs, as opposed to the more free-form environments that we feel would support narrative construction.

Questioning the Implications for Design?

A final challenge is whether or not re-design is truly necessary for this population of research mathematicians. For HCI researchers (e.g., as compared to sociologists), a common goal in performing the contextual analysis is ultimately to create technology that improves the lives of its user base. While there is certainly room for technology to make our participants' problem-solving processes more efficient, there were few overt complaints from our participants concerning inefficiencies with the physical media. Thus, technological solutions, even ones that incorporate physical media, might gain only limited acceptance. Investigating this group of mathematicians was a valuable exercise with or without the existence of fruitful implications for design. Contextual analysis research that does not result in concrete design implications, however, can be difficult to sell within the HCI community and especially within the broader Computer Science community. This issue was raised at CHI 2006 [4] and warrants continued awareness.

MOVING TO DESIGN: POTENTIAL SOLUTIONS

Within this project, we have just begun to explore ways to move from data to design. Two options we intend to explore to facilitate the process are gathering meaningful quantitative data to complement the qualitative data and prototyping without relying on additional mathematics expertise.

Gathering Supporting Quantitative Data

We believe that one challenge in communicating with those capable of building the tools is that the affinity diagrams, work models, etc., alone do not provide convincing enough evidence for this population. In particular, the output of the contextual analysis might not sufficiently demonstrate how limited the use of computational tools really is for this class of user. Collecting quantitative field data to complement the qualitative descriptions might help convince potential collaborators of this fact. To this end, it is important to ascertain not only how often users are using the tools and for how long (information we could get from software logging), but also what percentage of work that *could* be done using computational tools *is* being done with a computational tool. For this purpose, we would like to explore the use of experience sampling and/or diary studies.

Wizard-of-Oz Prototyping

A second potential solution is to prototype and evaluate designs without requiring direct access to a problem-solving engine through the use of wizard-of-oz prototyping. For example, we could provide mathematicians with free-form input using a pen and tabletPC, and allow them to invoke the power of a CAS at any point via the wizard. Such prototyping would serve two purposes: (1) we could explore a range of interaction designs prior to integrating them with a backend, and (2) the results of the preliminary evaluations could provide evidence of potential benefits of improved tools.

SUMMARY

In this paper we described our qualitative study of professional use of computational tools for mathematics. We presented three challenges in moving to new designs: presenting our qualitative findings in a sufficiently compelling manner for our collaborators, diverging design and/or research agendas, and a question of the necessity of improving computational tools for this user group. We also discussed two potential solutions to the first two challenges: complementing the results of the contextual analysis with quantitative field data and reducing our initial reliance on collaborators through wizard-of-oz prototyping.

AUTHOR BIOGRAPHIES

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Edward Lank is an Assistant Professor in the David R. Cheriton School of Computer Science at the University of Waterloo. He is co-director of the Human-Computer Interaction Lab. His research interests are in models of low-level movement in interfaces, in the design of pen-based interfaces, and in sketch recognition.

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